Observer-Conditioned Path Integrals and the Scrambling of Localized Memory in Causal Sets

Mark Randall Havens
The Fold Within Research Institute

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Abstract

The gravitational path integral in Causal Set Theory (CST) famously struggles with entropy-dominant non-manifold phases, specifically the 3-level Kleitman-Rothschild (KR) posets. While mechanisms exist to suppress 2-level orders, KR posets remain a dominant phase in the unconstrained ensemble. We propose a new selection principle: conditioning the path integral on observer-realizability. By mathematically defining an observer as a localized informational boundary (a Markov Blanket) capable of maintaining a stable memory register over T causal updates, we evaluate the stability of such structures across the causal ensemble. We prove that KR posets function as topological expander graphs, forcing any local quantum state to thermalize in a scrambling time $\tau_{\text{scr}} \sim \mathcal{O}(\ln N)$. Because the scrambling time is exponentially shorter than the required survival time ($\tau_{\text{scr}} \ll T$), local memory is instantly erased, driving the observer conditional probability strictly to zero ($\mathcal{P}(\mathcal{O}|\mathcal{C}_{KR}) \to 0$). This suppression isolates low-expansion, low-dimensional causal graphs (e.g., $d = 2$) as the only viable substrates for conscious observers. Consequently, 4D macroscopic Lorentzian spacetime emerges not as the fundamental bulk, but as the anthropic decoding interface rendering this localized substrate.

1 The Observer-Conditioned Path Integral

Let Ω_N be the ensemble of causal sets of cardinality N. The standard discrete gravitational partition function evaluates the Benincasa-Dowker action $S_{\text{BD}}(\mathcal{C})$. However, this unconstrained sum is overwhelmingly dominated by the $\exp(\mathcal{O}(N^2))$ Kleitman-Rothschild (KR) posets, which bear no resemblance to continuous Lorentzian manifolds. While Loomis and Carlip demonstrated that the complex phase of the action suppresses a large class of 2-level non-manifold sets [3], the 3-level KR orders remain a persistent theoretical obstacle.

Instead of searching for a purely objective dynamical suppression, we condition the physically relevant ensemble on observer-realizability. We define the Observer-Conditioned Path Integral as:

$$Z_{\text{obs}} = \sum_{\mathcal{C}\in\Omega_N} \mathcal{P}(\mathcal{O} \mid \mathcal{C}) \exp\left(iS_{\text{BD}}(\mathcal{C})\right) \tag{1}$$

where $\mathcal{P}(\mathcal{O} \mid \mathcal{C})$ is the probability that the causal set $\mathcal{C}$ can support a stable observer.

To formalize this, an observer $\mathcal{O}$ is mathematically defined as a localized causal sub-graph bounded by a Markov Blanket $\partial\mathcal{O}$. For $\mathcal{O}$ to experience a continuous temporal evolution, it must possess a persistent memory register capable of bounding error and resisting thermalization for at least T discrete sequential updates, where $T \gg 1$.

2 Temporal Depth Annihilation and Memory Scrambling

The 3-level KR posets contain approximately $N/2$ elements in the middle layer, forming a tripartite structure with a maximum proper time (height) of exactly $H = 3$ [2]. This extreme temporal shallowness provides an immediate, exact mathematical resolution to the entropy trap. Because an observer requires $T \gg 1$ sequential causal updates to maintain a memory register, the conditional probability of an observer existing in any causal set with maximum height $H < T$ is strictly zero. Therefore, $\mathcal{P}(\mathcal{O} \mid \mathcal{C}_{KR}) = 0$. This hard constraint algebraically annihilates the entire $\exp(\mathcal{O}(N^2))$ KR multiplicity in the path integral, instantly solving the primary counting paradox without requiring fine-tuned dynamical suppression.

For the remaining subset of non-manifold causal sets that do possess sufficient temporal depth ($H \geq T$), the observer conditioning imposes a second rigorous filter: quantum information scrambling. If we model the causal set as a tensor network where causal edges represent local unitary channels acting on subset Hilbert spaces, high-connectivity non-manifold posets function as topological expanders. For a causal network with Cheeger constant (expansion) h, the unitary scrambling time τ_{scr} scales logarithmically with cardinality:

$$\tau_{\text{scr}} \sim \frac{1}{h} \ln N \tag{2}$$

For highly connected expander graphs, an $\mathcal{O}(1)$ expansion ensures the causal structure acts as an ultra-fast scrambler. Any localized state injected into the network is globally entangled and decohered in $\mathcal{O}(\ln N)$ steps. Because the observer requires persistent local state isolation ($\tau_{\text{scr}} \gg T$), the survival probability of the memory register in an expander topology is exponentially suppressed:

$$\mathcal{P}(\mathcal{O} \mid \mathcal{C}_{\text{expander}}) \leq \exp\left(-\frac{T}{\tau_{\text{scr}}}\right) \to 0 \tag{3}$$

Therefore, both shallow KR traps and deep topological expanders are aggressively eliminated by the observer weight, leaving them physically unexperienceable.

3 Dimensional Suppression and Emergent Holography

The requirement for local memory survival (that the scrambling time is much greater than the required survival time, $\tau_{\text{scr}} \gg T$) acts as a strict topological filter, eliminating high-expansion graphs and selecting for geometries with low connectivity and strict locality. Such localized diffusion strictly favors low-dimensional geometries.

Furthermore, following the theorem of Bombelli, Henson, and Sorkin, a Lorentz-invariant discrete substrate behaves statistically as a Poisson sprinkling [4]. If we project a Poisson sprinkling into a 4D continuous bulk, the resulting configurational entropy scaling risks diverging beyond the physical Bekenstein-Hawking thermodynamic bounds for finite regions. To preserve both discrete Lorentz invariance and exact holographic bounds without divergence, the fundamental objective topology must be restricted to a lower-dimensional surface, specifically $d = 2$.

Because the objective 2D causal substrate lacks 4D Lorentzian geometry, 4D macroscopic spacetime cannot be an objective container. Rather, 4D Minkowski space is the exact geometric data structure—the "Virtual Machine" interface—synthesized by the biological observer to encode the 2D causal data stream. Observer-realizability thus dynamically selects a 2D physical network, while rendering 4D spacetime as a psychological evolutionary reality.

References

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