Observer-Conditioned Path Integrals and the Scrambling of Localized Memory in Causal Sets

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Abstract

The gravitational path integral in Causal Set Theory is pathologically dominated by highly connected, 3-level Kleitman-Rothschild (KR) posets, which overwhelm the manifold-like configurations required to recover classical spacetime. Rather than seeking purely dynamical suppression, we introduce an observer-conditioned selection principle. We demonstrate that KR posets and non-manifold expander graphs are fundamentally incompatible with the existence of localized observers. By requiring the persistence of a local memory register over a macroscopic timeline, the observer-conditioned partition function algebraically annihilates the $\exp(\mathcal{O}(N^2))$ KR multiplicity due to its insufficient temporal depth ($H = 3$). Furthermore, high-connectivity non-manifold posets function as topological expanders that rapidly scramble local quantum information in $\mathcal{O}(\ln N)$ steps, preventing memory survival. This strict observer-realizability constraint dynamically selects for low-dimensional, low-expansion causal substrates. We conclude by offering an ontological interpretation where 4D macroscopic Lorentzian spacetime emerges not as the objective bulk, but as the anthropic decoding interface (Virtual Machine) required to render the selected low-dimensional substrate.

1 Formalizing the Causal Observer

Let Ω_N be the ensemble of causal sets (locally finite partially ordered sets) of cardinality N. The standard discrete gravitational partition function evaluates the Benincasa-Dowker action $S_{\mathrm{BD}}(\mathcal{C})$ [1]. However, this unconstrained sum $\sum_{\mathcal{C}\in\Omega_N}$ is overwhelmingly dominated by the $\exp(\mathcal{O}(N^2))$ Kleitman-Rothschild (KR) posets [8]. While Loomis and Carlip demonstrated that the complex phase of the action suppresses a large class of 2-level non-manifold sets [6], the 3-level KR orders remain a persistent theoretical obstacle.

Rather than searching for a purely objective dynamical suppression, we introduce an exact algebraic filter by conditioning the physically relevant ensemble on observer-realizability. To formalize this, we construct a topological definition of an observer within a discrete order-theoretic framework.

Definition 1 (The Causal Observer): An observer $\mathcal{O}$ is defined as a localized causal sub-graph $\mathcal{O} = (V_{\mathcal{O}}, E_{\mathcal{O}})$ embedded within a global causal set $\mathcal{C} = (V, E)$, such that $V_{\mathcal{O}} \subset V$.

Definition 2 (The Causal Markov Blanket): The boundary of the observer is defined strictly by its order-theoretic causal relations. The Causal Markov Blanket $\partial\mathcal{O}$ is the union of the immediate causal past $J^{-}(V_{\mathcal{O}}) \setminus V_{\mathcal{O}}$ and causal future $J^{+}(V_{\mathcal{O}}) \setminus V_{\mathcal{O}}$ that directly interconnects $\mathcal{O}$ to the bulk environment $\mathcal{C} \setminus \mathcal{O}$.

Definition 3 (The Memory Register): For the observer $\mathcal{O}$ to experience continuous temporal evolution, it must possess an internal state space $\mathcal{H}_{\text{mem}}$ (a memory register). We mandate that this register must survive coherently for a macroscopic number of sequential updates. Topologically, this requires the existence of a causal chain (a totally ordered subset) within $V_{\mathcal{O}}$ of minimum length T, where $T \gg 1$.

2 The Observer-Conditioned Measure and KR Exclusion

We define the Observer-Conditioned Path Integral by restricting the sum over the observer-compatible subspace $\Omega_{\text{obs}} \subset \Omega_N$:

$$Z_{\text{obs}} = \sum_{\mathcal{C} \in \Omega_{\text{obs}}} \exp\left(iS_{\text{BD}}(\mathcal{C})\right) \tag{1}$$

where Ω_{obs} is the strict subset of causal sets that satisfy the conditions of Definitions 1-3. We can enforce this via a projection operator $\Pi_{\mathcal{O}}(\mathcal{C})$ such that $Z_{\text{obs}} = \sum_{\mathcal{C} \in \Omega_N} \Pi_{\mathcal{O}}(\mathcal{C}) \exp\left(iS_{\text{BD}}(\mathcal{C})\right)$.

This formulation allows us to prove the exact suppression of the entropy trap.

Proposition 1 (Temporal Depth Annihilation): The probability of a Kleitman-Rothschild poset $\mathcal{C}_{\text{KR}}$ supporting an observer $\mathcal{O}$ is strictly zero: $\Pi_{\mathcal{O}}(\mathcal{C}_{\text{KR}}) = 0$.

Proof. A Kleitman-Rothschild poset is defined as a tripartite 3-level order containing approximately $N/2$ elements in the middle layer [5]. By definition, the maximum proper time (chain length or height) of any $\mathcal{C}_{\text{KR}}$ is precisely $H = 3$. According to Definition 3, an observer requires a causal chain of minimum length $T \gg 1$. Since $H < T$, no continuous sub-graph satisfying Definition 3 can be embedded in $\mathcal{C}_{\text{KR}}$. Therefore, $\mathcal{C}_{\text{KR}} \notin \Omega_{\text{obs}}$. $\square$

This proposition algebraically annihilates the entire $\exp(\mathcal{O}(N^2))$ KR multiplicity from the physical path integral without requiring fine-tuned dynamical suppression.

3 Tensor Networks and Scrambling-Time Exclusion

For the remaining subset of non-manifold causal sets that possess sufficient temporal depth ($H \geq T$), the observer conditioning imposes a second rigorous filter based on quantum information dynamics.

To evaluate memory coherence, we map the discrete partial order to a tensor network. Causal links E are modeled as local unitary channels acting on the state spaces associated with the causal nodes. In graph-theoretic terms, high-connectivity non-manifold posets function as topological expander graphs.

Applying the fast-scrambling conjecture [7] to the graph-theoretic expansion (Cheeger constant) h of the poset's Hasse diagram, we model the unitary scrambling time τ_{scr} as scaling logarithmically with cardinality:

$$\tau_{\text{scr}} \sim \frac{1}{h} \ln N \tag{2}$$

Proposition 2 (Expander Scrambling Exclusion): Highly connected non-manifold causal sets (expander graphs) cannot support persistent localized memory.

Proof. For expander graphs, the Cheeger constant $h \sim \mathcal{O}(1)$, ensuring the causal structure acts as an ultra-fast scrambler. Any localized state in $\mathcal{H}_{\text{mem}}$ injected into the network is globally entangled and decohered in $\mathcal{O}(\ln N)$ steps. Because the observer requires persistent local state isolation ($\tau_{\text{scr}} \gg T$), and for these graphs $\tau_{\text{scr}} < T$ for physical memory bounds, expander topologies are excluded from the observer-compatible subspace Ω_{obs}. $\square$

Therefore, both shallow KR traps and deep topological expanders are exactly eliminated by the observer projection operator $\Pi_{\mathcal{O}}$, leaving them physically unexperienceable.

4 Dimensional Suppression and Holographic Bounds

The requirement for local memory survival ($\tau_{\text{scr}} \gg T$) acts as a strict topological filter, eliminating high-expansion graphs and selecting for geometries with low connectivity and strict locality.

Furthermore, following the theorem of Bombelli, Henson, and Sorkin, a Lorentz-invariant discrete substrate behaves statistically as a Poisson sprinkling [2]. However, the unconstrained sprinkling of discrete elements into a macroscopic 4D bulk generates a configurational entropy that scales with the bulk volume, violating the covariant holographic entropy bound [3], which requires entropy to scale with boundary area.

Proposition 3 (Holographic Dimensionality Bound): To preserve discrete Lorentz invariance while strictly satisfying covariant holographic entropy bounds, the selected physical substrate must be restricted to a lower-dimensional network ($d \leq 2$).

Proof (Sketch). If the causal substrate generates continuous 4D spacetime, its discrete elements must satisfy the Bousso bound $S \leq A/4G$, where A is the area of the bounding surface. A 4D Poisson sprinkling yields an extensive entropy $S \propto V_{4D}$. To prevent $V_{4D} > A$, the fundamental discrete graph cannot densely pack a 4D bulk; it must reside on a dimensionally reduced holographic screen ($d \leq 2$) such that its degrees of freedom scale consistently with the boundary area of the emergent spacetime. □

5 Interpretational Outlook: The Virtual Machine

Because the objective causal substrate is mathematically constrained to low-dimensional, low-expansion topologies ($d \leq 2$), 4D macroscopic Lorentzian spacetime cannot be an objective bulk container. Drawing on the interface theory of perception [4], we propose the ontological interpretation that 4D Minkowski space acts as an exact geometric data structure—a "Virtual Machine" interface—synthesized by the biological observer to decode the 2D causal data stream.

6 Conclusion

By conditioning the causal set path integral on observer-realizability via the projection operator $\Pi_{\mathcal{O}}$, we introduce an exact algebraic filter that eliminates the Kleitman-Rothschild entropy trap. The strict requirement for temporal depth ($H \geq T$) instantly zeroes the probability of $\exp(\mathcal{O}(N^2))$ shallow posets, while the fast-scrambling conjecture eliminates deep expander networks. This restricts the path integral to low-dimensional holographic substrates as the sole mathematically viable structures capable of supporting conscious observers. Future work will formalize the projection operators $\Pi_{\mathcal{O}}$ required to explicitly derive the 4D Virtual Machine geometry from this lower-dimensional state.

References

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