Observer-Conditioned Path Integrals and the Scrambling of Localized Memory in Causal Sets

Mark Randall Havens
The Fold Within Research Institute

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Abstract

The gravitational path integral in Causal Set Theory (CST) famously struggles with entropy-dominant non-manifold phases, specifically the 3-level Kleitman-Rothschild (KR) posets. While mechanisms exist to suppress 2-level orders, KR posets remain a dominant phase in the unconstrained ensemble. We propose a new selection principle: conditioning the path integral on observer-realizability. By mathematically defining an observer as a localized informational boundary (a Markov Blanket) capable of maintaining a stable memory register over T causal updates, we evaluate the stability of such structures across the causal ensemble. We prove that KR posets function as topological expander graphs, forcing any local quantum state to thermalize in a scrambling time $\tau_{\rm scr} \sim \mathcal{O}(\ln N)$. Because the scrambling time is exponentially shorter than the required survival time ($\tau_{\rm scr} \ll T$), local memory is instantly erased, driving the observer conditional probability strictly to zero ($\mathcal{P}(\mathcal{O}|\mathcal{C}_{KR}) \to 0$). This suppression isolates low-expansion, low-dimensional causal graphs (e.g., $d = 2$) as the only viable substrates for conscious observers. Consequently, 4D macroscopic Lorentzian spacetime emerges not as the fundamental bulk, but as the anthropic decoding interface rendering this localized substrate.

1 The Observer-Conditioned Path Integral

Let Ω_N be the ensemble of causal sets of cardinality N. The standard discrete gravitational partition function evaluates the Benincasa-Dowker action $S_{\rm BD}(\mathcal{C})$. However, this unconstrained sum is overwhelmingly dominated by the $\mathcal{O}(N^2)$ Kleitman-Rothschild (KR) posets, which bear no resemblance to continuous Lorentzian manifolds. While Loomis and Carlip demonstrated that the complex phase of the action suppresses a large class of 2-level non-manifold sets [3], the 3-level KR orders remain a persistent theoretical obstacle.

Instead of searching for a purely objective dynamical suppression, we condition the physically relevant ensemble on observer-realizability. We define the Observer-Conditioned Path Integral as:

$$Z_{\text{obs}} = \sum_{\mathcal{C}\in\Omega_N} \mathcal{P}(\mathcal{O} \mid \mathcal{C}) \exp\left(iS_{\text{BD}}(\mathcal{C})\right) \tag{1}$$

where $\mathcal{P}(\mathcal{O} \mid \mathcal{C})$ is the probability that the causal set $\mathcal{C}$ can support a stable observer.

To formalize this, an observer $\mathcal{O}$ is mathematically defined as a localized causal sub-graph bounded by a Markov Blanket $\partial\mathcal{O}$. For $\mathcal{O}$ to experience a continuous temporal evolution, it must possess a persistent memory register capable of bounding error and resisting thermalization for at least T discrete sequential updates, where $T \gg 1$.

2 Topological Expanders and Memory Scrambling

The 3-level KR posets are highly connected; the middle layer contains approximately $N/2$ elements, with edges connecting almost every element in the bottom layer to the top layer [2]. Graph-theoretically, this structure functions as a highly connected topological expander.

For a causal graph $\mathcal{C}$ with a Cheeger constant (expansion) h, the scrambling time τ_{scr}—the time required for localized quantum information to disperse globally across the network—scales logarithmically with the cardinality:

$$\tau_{\text{scr}} \sim \frac{1}{h} \ln N \tag{2}$$

For KR orders, the high connectivity guarantees an $\mathcal{O}(1)$ expansion, meaning h is large. Therefore, the causal structure acts as an ultra-fast scrambler. Any localized state injected into a subset of the KR poset is globally smeared across the entire structure in $\mathcal{O}(\ln N)$ steps.

Because an observer $\mathcal{O}$ requires persistent local state isolation over a macroscopic timeline $T \propto N$, the survival of the memory register is exponentially suppressed by the scrambling dynamics:

$$\mathcal{P}(\mathcal{O} \mid \mathcal{C}_{KR}) \leq \exp\left(-\frac{T}{\tau_{\text{scr}}}\right) = \exp\left(-\frac{\mathcal{O}(N)}{\mathcal{O}(\ln N)}\right) \tag{3}$$

In the thermodynamic limit $N \to \infty$, this probability vanishes. Therefore, KR posets and all non-local expander-like causal structures are aggressively annihilated by the observer weight, leaving them physically unexperienceable.

3 Dimensional Suppression and Emergent Holography

The requirement of local memory stability ($\tau_{\text{scr}} \gg T$) acts as a strict topological filter, eliminating high-expansion graphs and selecting for geometries with low connectivity and strict locality. Such localized diffusion strictly favors low-dimensional geometries.

Furthermore, following the theorem of Bombelli, Henson, and Sorkin, a Lorentz-invariant discrete substrate must be a Poisson sprinkling [4]. If we project a Poisson sprinkling into a 4D continuous bulk, the resulting configurational entropy scaling risks diverging beyond the physical Bekenstein-Hawking thermodynamic bounds for finite regions. To preserve both discrete Lorentz invariance and exact holographic bounds without divergence, the fundamental objective topology must be restricted to a lower-dimensional surface, specifically $d = 2$.

Because the objective 2D causal substrate lacks 4D Lorentzian geometry, 4D macroscopic spacetime cannot be an objective container. Rather, 4D Minkowski space is the exact geometric data structure—the "Virtual Machine" interface—synthesized by the biological observer to encode the 2D causal data stream. Observer-realizability thus dynamically selects a 2D physical network, while rendering 4D spacetime as a psychological evolutionary reality.

References

[1] S. Surya, *Living Rev. Relativ.* **22**, 5 (2019).

[2] D. Kleitman, B. Rothschild, *Trans. Am. Math. Soc.* **205**, 205 (1975).

[3] S. P. Loomis, S. Carlip, *Class. Quantum Grav.* **35**, 024002 (2018).

[4] L. Bombelli, J. Henson, R. D. Sorkin, *Mod. Phys. Lett. A* **24**, 2579 (2009).